Type Theory for Weak ω-categories

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The `Catt` prover : Motivations

- Describing the theory of weak ω-categories is very difficult : infinitly many operations and axiomes
- `Catt` is a prover for this theory. It is a typechecker : terms are operations, they typecheck if and only if they are well defined.
- Many examples will be shown both as illustration of the theory and to explain the syntax

Outline

Weak ω-categories their proof theory

Type theory for weak ω-categories

Categories

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- A set of points $\mathcal{C}_0 = \{A, B, ...\}$

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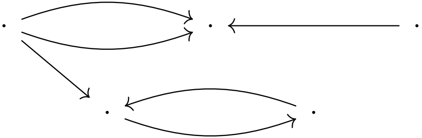

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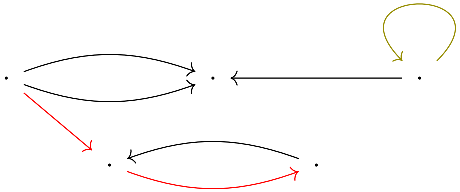
- With

$$\begin{cases} \textit{composition} & \circ : \{g, f \in \mathcal{C}_1, t(f) = s(g)\} \to \mathcal{C}_1 \\ \textit{identities} & \forall A \in \mathcal{C}_0, \mathrm{id}_A \in \mathcal{C}_1 \end{cases}$$



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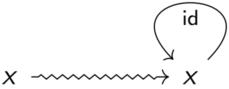
```
coh id (x : *) : * | x -> x
```



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```
coh comp (x : *) (y : *) (f : * | x -> y)
         (z : *) (g : * | y -> z) : * | x -> z
```

$$x \xrightarrow{f} y \xrightarrow{g} z \rightsquigarrow x \xrightarrow{\text{comp}} z$$

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```
coh wrong (x : *) (y : *) (f : * | x -> y)
          (z : *) (t : *) (g : * | z -> t) : * |x -> t
```

$$x \xrightarrow{f} y$$

$$z \xrightarrow{g} t$$

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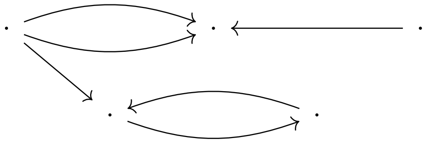
- Satisfying $\begin{cases} \textit{associativity} & h \circ (g \circ f) = (h \circ g) \circ f \\ \textit{unit law} & f \circ \mathrm{id} = \mathrm{id} \circ f = f \end{cases}$



2-categories

A 2-category $\mathcal{C}$ is

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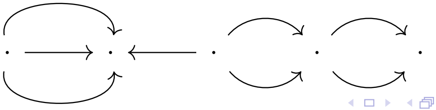

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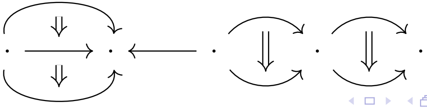

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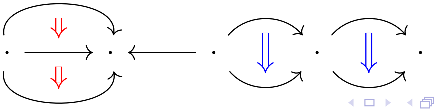
$$\begin{cases} \textit{vertical composition} & \circ : \{\beta, \alpha \in \mathcal{C}_2, t(\alpha) = s(\beta)\} \to \mathcal{C}_2 \\ \textit{horizontal composition} & \star : \{\beta, \alpha \in \mathcal{C}_2, tt(\alpha) = ss(\beta)\} \to \mathcal{C}_2 \\ \textit{identities} & \forall f \in \mathcal{C}_1, \mathrm{id}_f \in \mathcal{C}_2 \end{cases}$$



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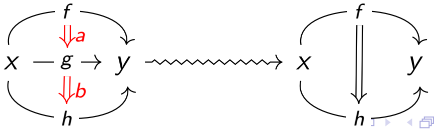
```
coh vcomp (x : *) (y : *) (f : * | x -> y)
          (g : * | x -> y) (a : * | x -> y | f -> g)
          (h : * | x -> y) (b : * | x -> y | g -> h)
          : * | x -> y | f -> h
```



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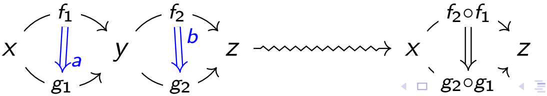
```
coh hcomp (x : *) (y : *) (f1 : * | x -> y)
          (g1 : * | x -> y) (a : * | x -> y | f -> g)
          (z : *) (f2 : * | y -> z)
          (g2 : * | y -> z) (b : * | y -> z | f2 -> g2)
    : * | x -> z | comp [x y f1 z f2] -> comp [x y g1 z g2]
```



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- Satisfying $\begin{cases} \textit{associativities} & \gamma \circ (\beta \circ \alpha) = (\gamma \circ \beta) \circ \alpha \\ & \gamma \star (\beta \star \alpha) = (\gamma \star \beta) \star \alpha \\ \textit{unit laws} & \alpha \circ \mathrm{id} = \mathrm{id} \circ \alpha = \alpha \\ & \alpha \star \mathrm{id} = \mathrm{id} \star \alpha = \alpha \end{cases}$

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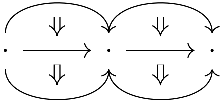
- exchange law $(\alpha \star \beta) \circ (\gamma \star \delta) = (\alpha \circ \gamma) \star (\beta \circ \delta)$



Climbing in dimension

- ω-categories are obtained by adding 3-morphisms between the 2-morphisms, then 4-mrophisms between the 3-morphisms, and so on

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- ω-categories are obtained by adding 3-morphisms between the 2-morphisms, then 4-mrophisms between the 3-morphisms, and so on
- Every time, there are parallelism conditions, compositions and axioms

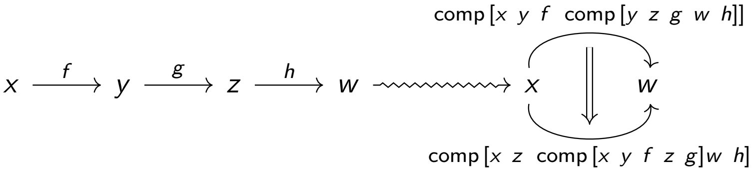
Weak ω-categories

- How to encode the axioms of an ω-categor internally?

Weak ω-categories

- How to encode the axioms of an ω-categor internally?
- $n+1$-morphisms are very natural candidates to encode axioms concerning n-morphisms

Example :
associativity : there exists an invertible 2-morphism $h \circ (g \circ f) \Rightarrow (h \circ g) \circ f$



Weak ω-categories

- How to encode the axioms of an ω-categor internally?
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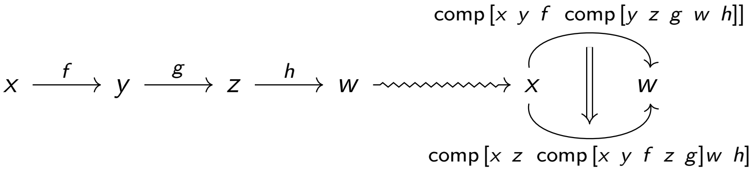
Example :

```
coh assoc (x : *) (y : *) (f : * | x -> y)
                   (z : *) (g : * | y -> z)
                   (w : *) (h : * | z -> t)
 : * | x -> t | comp[x y f w comp[y z g w h]] ->
                comp[x z comp[x y f z g] w h]
```



Weak ω-categories

- How to encode the axioms of an ω-categor internally?
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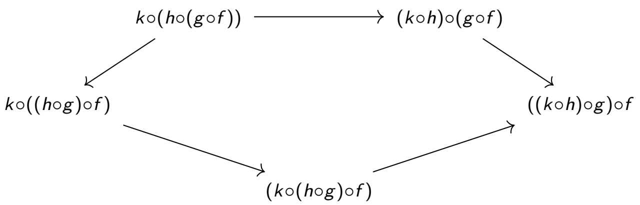
- We then have to encode the coherences between these coherences

Example MacLane's Pentagon



Weak ω-categories

- How to encode the axioms of an ω-categor internally?
- $n+1$-morphisms are very natural candidates to encode axioms concerning n-morphisms
- We then have to encode the coherences between these coherences
- We get the weak ω-categories

Adding all compositions

Key idea : do the compositions in the most general way possible

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Examples :

```
coh comp3 (x : *) (y : *) (f : * | x -> y)
          (z : *) (g : * | y -> z)
          (w : *) (h : * | z -> w)
        : * | x -> w
```

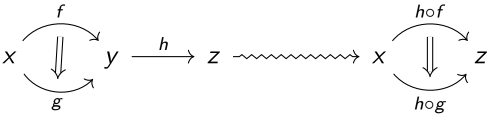
$$x \xrightarrow{f} y \xrightarrow{g} z \xrightarrow{h} w \rightsquigarrow x \longrightarrow w$$

Adding all compositions

Key idea : do the compositions in the most general way possible

Examples :

```
coh whisker (x : *) (y : *) (f : * | x -> y)
            (g : * | x -> y) (a : * | x -> y | f -> g)
            (z : *) (h : * | y -> z)
  : * | x -> z | comp [x y f z h] -> comp [x y g z h]
```



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type theory for globular sets

- The combinatorics of points, arrows and higher dimensional arrows

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- Typing rules :

$$\frac{\Gamma \vdash}{\Gamma \vdash \star} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash x : A \qquad \Gamma \vdash y : A}{\Gamma \vdash x \underset{A}{\to} y}$$

type theory for globular sets

- The combinatorics of points, arrows and higher dimensional arrows
- Typing rules :

$$\frac{\Gamma \vdash}{\Gamma \vdash \star} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash x : A \qquad \Gamma \vdash y : A}{\Gamma \vdash x \underset{A}{\to} y}$$

Examples :

- $\varnothing \vdash \star$

  Objects defined without any assumption
- $(x : \star, y : \star) \vdash x \underset{\star}{\to} y$

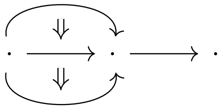
  Morphisms defined between any two objects
- $(x : \star, y : \star, f : x \underset{\star}{\to} y, g : x \underset{\star}{\to} y) \vdash f \underset{x \underset{\star}{\to} y}{\to} g$

  2-morphisms defined between any two parallel morphisms
- No derivation for 2-morphisms between two non-parallel morphisms

Pasting schemes

- Pasting schemes : a way to index all operations and axioms of weak ω-categories
  They are configurations of points and morphisms that can be composed altogether

Examples :



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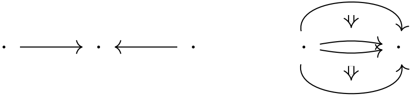
Examples :

Counter-examples :



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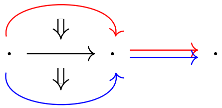

- Pasting also have sources and targets

Pasting schemes

- Pasting schemes : a way to index all operations and axioms of weak ω-categories
  They are configurations of points and morphisms that can be composed altogether

Examples :

- Pasting also have sources and targets
- In type theory :
  Judgement $\Gamma \vdash_{\mathsf{ps}}$ holding if and only if the diagram corresponding to Γ is a pasting scheme. Source and target computables

Adding the compositions...

Compositions are handled with new term constructor

Adding the compositions...

$$\frac{\Gamma \vdash_{\mathsf{ps}} \qquad \Gamma \vdash t \underset{A}{\to} u \qquad \partial^-\Gamma \vdash t : A \qquad \partial^+\Gamma \vdash u : A}{\Gamma \vdash \mathsf{coh}_{\Gamma,A} : t \underset{A}{\to} u}$$

$$\text{whenever } \left\{ \begin{array}{rcl} Var(t) & = & Var(\partial^-(\Gamma)) \\ Var(u) & = & Var(\partial^+(\Gamma)) \end{array} \right.$$

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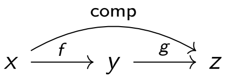
$$\text{whenever} \begin{cases} Var(t) = Var(\partial^-(\Gamma)) \\ Var(u) = Var(\partial^+(\Gamma)) \end{cases}$$

Examples :

- The composition of morphisms

$$(x : \star, y : \star, f : x \underset{\star}{\to} y, z : \star, g : y \underset{\star}{\to} z) \vdash \text{comp} : x \underset{\star}{\to} z$$



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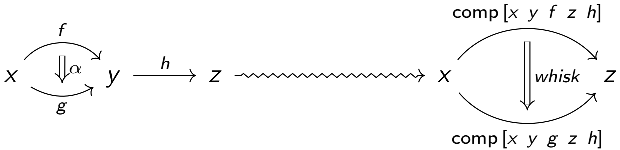
$$\text{whenever} \begin{cases} Var(t) = Var(\partial^-(\Gamma)) \\ Var(u) = Var(\partial^+(\Gamma)) \end{cases}$$

Examples :

- The "whiskering" : if
  $\Gamma = (x : \star, y : \star, f : x \underset{\star}{\to} y, g : x \underset{\star}{\to} y, \alpha : f \underset{x \underset{\star}{\to} y}{\to} g, z : \star, h : y \underset{\star}{\to} z)$

$$\Gamma \vdash whisk : \mathrm{comp}[x\ y\ f\ z\ h] \underset{x \underset{\star}{\to} z}{\to} \mathrm{comp}[x\ y\ g\ z\ h]$$



… And the axioms

Axioms are morphisms too! They are handled by term constructor

… And the axioms

$$\frac{\Gamma \vdash_{\mathsf{ps}} \qquad \Gamma \vdash A \qquad \Gamma \vdash t : A \qquad \Gamma \vdash u : A}{\Gamma \vdash \mathsf{coh}_{\Gamma, t \underset{A}{\to} u} : t \underset{A}{\to} u}$$

$$\text{whenever } \begin{cases} Var(t) = Var(\Gamma) \\ Var(u) = Var(\Gamma) \end{cases}$$

… And the axioms

$$\frac{\Gamma \vdash_{\mathrm{ps}} \qquad \Gamma \vdash A \qquad \Gamma \vdash t : A \qquad \Gamma \vdash u : A}{\Gamma \vdash \mathrm{coh}_{\Gamma, t \underset{A}{\to} u} : t \underset{A}{\to} u}$$

$$\text{whenever } \begin{cases} Var(t) = Var(\Gamma) \\ Var(u) = Var(\Gamma) \end{cases}$$

Examples :

- associativity
  $\Gamma = (x : \star, y : \star, f : x \underset{\star}{\to} y, z : \star, g : y \underset{\star}{\to} z, w : \star, h : z \underset{\star}{\to} w)$

$$\Gamma \quad : \quad x \xrightarrow{f} y \xrightarrow{g} z \xrightarrow{h} w$$

$$\Gamma \vdash \mathrm{ass} : \mathrm{comp}[x\ y\ f\ w\ \ \mathrm{comp}[y\ z\ g\ w\ h]] \underset{x \underset{\star}{\to} w}{\to} \mathrm{comp}[x\ z\ \ \mathrm{comp}[x\ y\ f\ z\ g] w\ h]$$

... And the axioms

$$\frac{\Gamma \vdash_{\mathrm{ps}} \qquad \Gamma \vdash A \qquad \Gamma \vdash t : A \qquad \Gamma \vdash u : A}{\Gamma \vdash \mathrm{coh}_{\Gamma, t \underset{A}{\to} u} : t \underset{A}{\to} u}$$

$$\text{whenever } \begin{cases} Var(t) = Var(\Gamma) \\ Var(u) = Var(\Gamma) \end{cases}$$

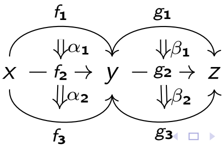
Examples :

- exchange law

$$\Gamma = (x:\star, y:\star, f_1 : x \underset{\star}{\to} y, f_2 : x \underset{\star}{\to} y, \alpha_1 : f_1 \underset{x \underset{\star}{\to} y}{\to} f_2, f_3 : x \underset{\star}{\to} y, \alpha_2 : f_2 \underset{x \underset{\star}{\to} y}{\to} f_3,$$

$$z:\star, g_1 : y \underset{\star}{\to} z, g_2 : y \underset{\star}{\to} z, \beta_1 : g_1 \underset{y \underset{\star}{\to} z}{\to} g_2, g_3 : y \underset{\star}{\to} z, \beta_2 : g_2 \underset{y \underset{\star}{\to} z}{\to} g_3)$$

Γ :



… And the axioms

$$\frac{\Gamma \vdash_{\mathrm{ps}} \qquad \Gamma \vdash A \qquad \Gamma \vdash t : A \qquad \Gamma \vdash u : A}{\Gamma \vdash \mathrm{coh}_{\Gamma, t \underset{A}{\to} u} : t \underset{A}{\to} u}$$

$$\text{whenever } \begin{cases} Var(t) = Var(\Gamma) \\ Var(u) = Var(\Gamma) \end{cases}$$

Examples :

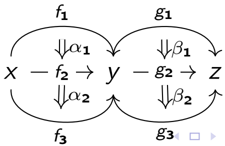
- exchange law

$\Gamma \vdash \mathrm{hcomp}[x\ \ y\ \ f_1\ \ f_3\ \ \mathrm{vcomp}[x\ \ y\ \ f_1\ \ f_2\ \ \alpha_1\ \ f_3 \alpha_2]\ \ z\ \ g_1\ \ g_3\, \mathrm{vcomp}[y\ \ z\ \ g_1\ \ g_2\ \ \beta_1\ \ g_3\ \ \beta_2]]$

$\overrightarrow{\mathbf{comp}[x\ \ y\ \ f_1\ \ z\ \ g_1] \underset{x \underset{\star}{\to} z}{\to} \mathbf{comp}[x\ \ y\ \ z\ \ f_3\ \ z\ \ g_3]}$

$\mathrm{vcomp}[x\ \ z\ \ \mathrm{comp}[x\ \ y\ \ f_1\ \ z\ \ g_1]\ \ \mathrm{comp}[x\ \ y\ \ f_2\ \ z\ \ g_2]\, \mathrm{hcomp}[x\ \ y\ \ f_1\ \ f_2 \alpha_1 z\ \ g_1\ \ g_2 \beta_1]$

$\mathrm{comp}[x\ \ y\ \ f_3\ \ z\ \ g_3]\, \mathrm{hcomp}[x\ \ y\ \ f_2\ \ f_3\ \ \alpha_2\ \ g_2\ \ g_3\ \ \beta_2]]$

$\Gamma \quad :$ 

Syntactic sugar

- Shortening types

Example :

$$(x : \star, y : \star) \vdash x \underset{\star}{\to} y \rightsquigarrow (x : \star, y : \star) \vdash x \to y$$

Syntactic sugar

- Shortening types
- Implicit arguments

Remark : Many arguments are redundant. We can give fewer terms, and decide which ones to give

Example :

$$\Gamma = (x : \star, y : \star, f : x \to y, z : \star, g : y \to z, w : \star, h : z \to w)$$

$$\Gamma \vdash \text{ass} : \text{comp}[x\ \ y\ \ f\ \ w\ \ \text{comp}[y\ \ z\ \ g\ \ w\ \ h]] \to \text{comp}[x\ \ z\ \ \text{comp}[x\ \ y\ \ f\ \ z\ \ g]\ \ w\ \ h]$$

$$\rightsquigarrow$$

$$\Gamma \vdash \text{ass} : \text{comp}[f\ \ \text{comp}[g\ \ h]] \to \text{comp}[\ \text{comp}[f\ \ g]\ \ h]$$

Syntactic sugar

- Shortening types
- Implicit arguments
- Parametricity by dimension

Remark : all operation and axiom is still valid if we increase all the dimensions of the arrow by the same number

Example :

$$\left.\begin{array}{l}(x:\star)\vdash \mathrm{id}_0 : x \to x \\ (x:\star, y:\star, f : x \to y)\vdash \mathrm{id}_1 : f \to f \\ \vdots\end{array}\right\} \rightsquigarrow (x:\star)\vdash \mathrm{id} : x \to x$$

Try it yourself!

`https://github.com/ThiBen/catt`